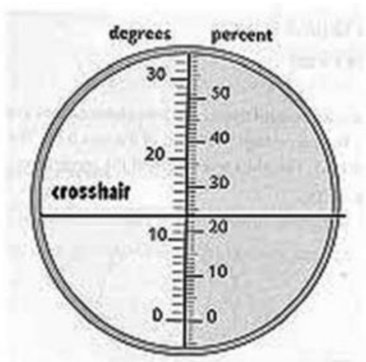
Using a clinometer, a measuring tape and basic trigonometry to determine distances on sloping sites

What is a clinometer? - A clinometer (or inclinometer) is an instrument for measuring angles of slope or percent grade (also known as % slope) of a distance with respect to gravity. Clinometers can measure both inclines (positive slopes), as seen by an observer looking upwards and declines (negative slopes), as seen by an observer looking downward. A clinometer used with a simple linear distance-measuring device, such as a measuring tape or laser distance-measurer (LDM) and the principles of trigonometry, allows determination of elevation change between two points. A clinometer and a distance-measuring device used with the principles of trigonometry also enable the conversion of a measured slope distance to the horizontal distance typically shown on a plat or site plan.

Types of clinometers – There are several different types of clinometers (electronic and optical) made by various manufacturers. The most important thing to know about the clinometer you are using is the unit(s) of measure used in the device. Most clinometers display both degrees and percent slope. One of the most commonly used models is the Suunto PM-5/360. The picture below shows what you should see, generally, when sighting through this particular



clinometer. This clinometer has the scale on the left calibrated in degrees and the scale on the right is calibrated in percent slope (% grade). While sighting through the clinometer, tilting it will cause the scales to move in the viewfinder to show the angle of the device. You can quickly check the units of measure for the clinometer you are using by tilting it until it reads 45º on the left hand scale at the crosshair. If the clinometer has degree and percent slope scales, it will simultaneously read 100% at the crosshair on right hand scale…100% slope is equivalent to a 45º angle. If your clinometer is not showing this, refer to the instructions that came with the device or find the clinometer manufacturer name and model number and look for the instructions online. Knowing the units of the clinometer scales and how to read them are essential for obtaining accurate results.

Distance Measuring Devices – Likewise, for the distance-measuring device you are using (tape measure or LDM), note the units of measurement. If the device measures in units of feet and tenths (as on an “engineers” scale) then the measurement is in decimal form (as on an engineer’s scale) and you can read the distance directly from the instrument. However, if the device measures in units of feet and inches, convert the measurement into decimal feet to simplify the math involved. For example, the decimal feet equivalent of a measurement of 71 feet, 9 inches is determined as follows;

71 ft + 9 in =71 ft + $\frac{9\ inches}{12\ inches\ per\ foot}$ *(or 0.75 feet)…So a distance of 71 feet 9 inches = 71 + 0.75 or 71.75 feet*

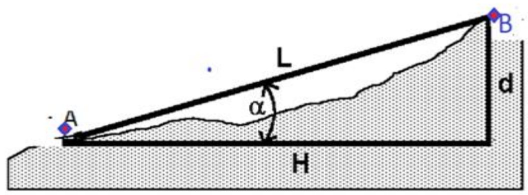
Some electronic laser distance-measuring devices measure yards or meters. If you are using one of these, be sure to convert the measured distance to feet to keep units consistent throughout the process and avoid errors. It is always a good idea to practice using your devices and become very familiar with them before using them in the field.

Converting feet measured along a slope to horizontal feet as scaled on a plat or site plan

Information on survey plats include horizontal distances of boundaries between property corners. In addition, plats or site diagrams may show other features such as drainage structures, wells, and buildings drawn to scale or dimensioned. The distances stated or shown on plats and site plans are always *horizontal feet* unless otherwise noted. However, when we measure distances on sloping sites we are typically measuring a sloped distance rather than a horizontal distance. Recognizing that there is a difference between horizontal distance and sloped distance is important. Conversion of sloped distances to horizontal distances is necessary when using or making scaled drawings, particularly on steeply sloping sites.

Depending upon the accuracy needed, slope distance may be a good estimate of horizontal distance, unless the slope is significant. The table at the right shows the percentage error resulting from an assumption that slope distance is equal to horizontal distance (H = L) at varying inclines (grade (%) and equivalent slope angle - degrees). Note the error of such an assumption is less than 2% at grades below 20% (slope angle of 11.3^o). *Horizontal distance is always less than the measured slope distance.* Accurately illustrating relative separations and setbacks (to wells, property lines, etc.) on a copy of a plat or site diagram that is drawn to scale, requires measured slope distances to distances be converted to horizontal distances when a *"significant"* slope is present. Again, the definition of *"significant"* depends upon the *accuracy needed* for the situation. For situations where accuracy is critical - such as when area is very limited or potable water sources are located in close proximity to wastewater system components - the consequences of errors may be great. In these cases consider obtaining a survey prepared by an appropriately licensed individual.

Percent error when assuming slope distance = horizontal length at varying grades and slope angles		
(% grade) $100\,x\frac{d}{H}$	**slope angle "α" (degrees)**	**% error H = L**
0	0.0	0%
10	5.7	0%
20	11.3	2%
30	16.7	4%
40	21.8	7%
50	26.6	11%
100	45.0	29%



Encountering a circumstance such as that represented by the diagram to the left, we can use a clinometer and measuring tape to determine the horizontal distance "H" between Point A and Point B. Tools from trigonometry make this possible and include the following relationships between the distances L, H and d and the angle "α" (in degrees).

$$\mathbf{d = L \times \sin\alpha} \qquad \mathbf{H = L \times \cos\alpha} \qquad \mathbf{L = \left(\frac{H}{\cos\alpha}\right)} \qquad \mathbf{\tan\alpha = \frac{d}{H}}$$

Step 1 *- Using the measuring tape, determine the "slope" distance* ***"L"*** *between Points A and B by pulling the tape taut, but taking care not to break the tape. Record the distance in terms of decimal feet. (see above for instructions on converting feet and inches to decimal feet) For this example, assume we have measured distance* ***L*** *to be 71.75 feet)*

Step 2 *– With the clinometer, measure the angle of inclination* ***("α")*** *of the slope along "****L****" in degrees or % slope. When tilting the clinometer, sight the distant point so that it is approximately the same elevation above the ground as your eye level when using the clinometer. (Consider using an auger in a shallow hole at the point sighted with the handle set at eye level as a guide for insuring equal elevations of the clinometer at eye level and the distant sighting point. Use the clinometer to sight the handle.) Record angle of inclination "****α****", or the percent slope of "****L****" between Point A and Point B. For this example assume we have approximated "****α****" = 13^o or 23% grade.*

Step 3 *– Using a scientific calculator (or a cosine table such as that on Page 4) determine H as follows:*

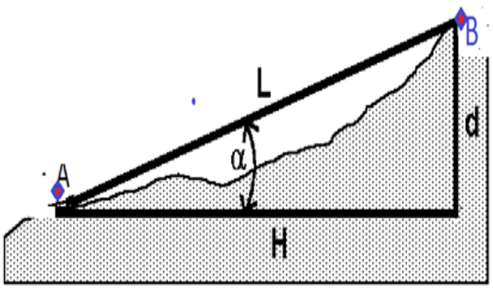
i. *When using "****α****" in degrees* ***H*** *=* ***L*** *x cosine(****α****) ; In our example* ***L*** *= 71.75 feet and* ***α*** *is measured as 13^o*
   H *= 71.75 ft x cosine (13^o) = 71.75 x 0.9744 = 69.9 ft = horizontal distance between Pts A & B*

ii. *If your clinometer only has a scale for % (grade) and doesn't give slope angle ("****α****") in degrees, first convert % slope to degrees using the following equation*
   *"****α****" in degrees =* $\tan^{-1}\left(\frac{\%\ grade}{100}\right)$*; …in this example the clinometer shows 23% slope, so*
   *"****α****" =* $\tan^{-1}\left(\frac{23}{100}\right) = \tan^{-1}(0.23) = 13^o$*; (rounded to nearest 0.5^o), …then proceed as in "i" above…*

H *= 71.75 ft x cosine (13^o) = 71.75 x 0.9744 = 69.9 ft = horizontal distance (H) between Pts A & B*

Scientific calculators will access trigonometric values for sine, cosine and tangents of angles. See the abbreviated trigonometry tables on Page 4 for sine, cosine and tangent values if a scientific calculator is unavailable.

(A note about calculators ... the function $\tan^{-1}$ is alternatively known as the "inverse tangent" or "arc tangent" function. "ATAN" is the label given this function on some calculator keypads. On others, the key says $\tan^{-1}$. To verify the mode of operation of your calculator, check to see that $\tan^{-1}$ (arctangent, inverse tangent or ATAN) of 0.231 is given as approximately 13^o on the calculator. If it is not, verify that the calculator is operating in the "degrees" rather than the "radians" mode. Refer to the instructions that came with the calculator.)



Similarly, you may use a clinometer and a measuring tape to determine the elevation difference between two points. This is essential information when specifying a pump to deliver water or wastewater to a higher elevation. Static or elevation head is the difference between the water level in the pump station and the discharge point of the force main (pump line). The following is a step procedure to determine the elevation difference between two points.

Using a Clinometer and Measuring Tape to Estimate the Elevation Difference "d" between Two Points

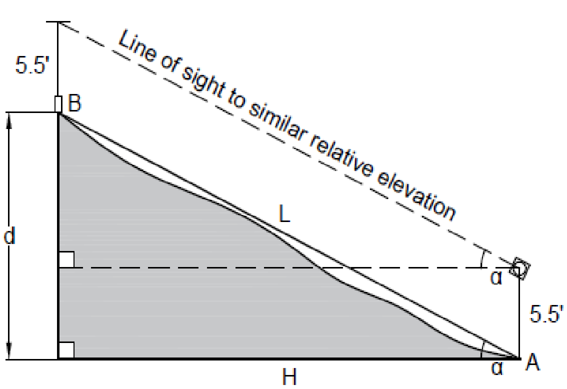
Step A *– Using the measuring tape, determine the distance* ***"L"*** *between Points A and B by pulling the tape as taut as possible while taking care not to break the tape. Record the distance in terms of decimal feet. (See above for instructions on converting feet and inches to decimal feet) For this example, assume we have measured distance L to be 71.75 feet.*

Step B *– With the clinometer, measure the angle of inclination ("**α**") of the slope along "**L**" in degrees or percent (%) grade. When tilting the clinometer, sight the distant point so that it is approximately the same elevation above the ground as your eye level when using the clinometer (when this is not possible, utilize Step D below to correct for the difference). Record angle of inclination* ***"α",*** *(or the percent slope) of "**L**" between Point A and Point B. For this example - assume we have approximated "**α**" = 13^o (23% grade) using the clinometer.*

Step C *– Determine the elevation difference* ***"d"*** *as follows:*

i. When using "**α**" in degrees **d** = **L** x sin(**α**) ; In our example **L** = 71.75 feet and **α** is measured as 13^o, so
H = 71.75 ft x sin (13^o) = 71.75 x 0.2250 = 16.1 ft = elevation (vertical) difference between Pts A & B

ii. If using **% slope** instead of "**α**" in degrees, *first convert % slope to degrees using the following equation:*



"α" *in degrees =* $\tan^{-1}\left(\frac{\%\ slope}{100}\right)$*...in this example the clinometer shows 23% slope, so* ***"α"*** = $\tan^{-1}\left(\frac{23}{100}\right)$ =

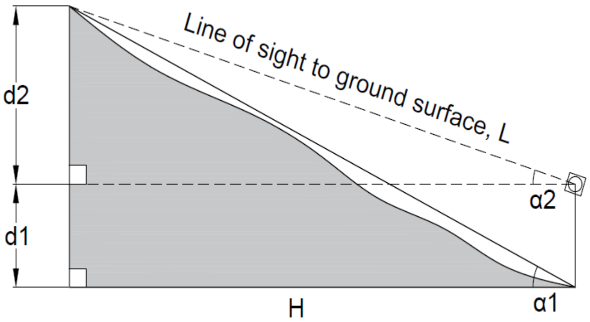
$\tan^{-1}$ (0.23) = 13^o; (rounded to nearest 0.5^o), …
then proceed as in "i" above;

"d" *= 71.75 ft x sin (13^o) = 71.75 x 0.2250 = 16 ft*

= elevation (vertical) difference between Pts A & B

Step D *– If the clinometer and the point sighted are at the same distance above ground, as shown on the left, this step (Step D Correction) is not necessary.*

Elevation (Height) Correction Required

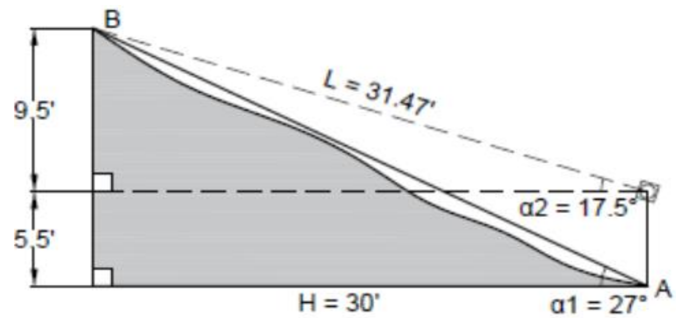


Note: α1 ≠ α2, α1 > α2.

Otherwise, sighting a distant point at an elevation different than that of the clinometer, (i.e. ground level) as shown on the left - requires correction of the elevation difference by adjusting for the difference in the clinometer elevation above ground at the observers' position and the elevation of the point above ground of the distant point sighted.

Here is an example. The observer in this instance is 5'10" tall and has an eye level of 5'6" (5.5 ft) from the ground. Assume it is impossible or inconvenient for the observer to set a sighting point at eye level, 5'6 above the ground at point B. Can the elevation difference from ground level at point A to the ground level at point B be determined? ***YES***

On the diagram at the right, standing at point A, the observer holds the clinometer at eye level, sighting a point on the ground at point B. The clinometer shows the slope angle "α2" = 17.5º (and /or a grade 31%). Using a tape-measure (or laser distance-measurer), the observer determines the distance along the slope to be 31.47 ft. (round to 31.5')



From the equation in Step C above: d2 = L x sine α2 = 31.5 feet x sine (17.5º) = 31.5 ft x 0.3007 = 9.5 feet

To this we must add the distance from the clinometer (eye level of the observer at Point A) to the ground surface at Point A (5.5 ft) to get the total elevation difference from Point A to Point B………. 9.5 ft + 5.5 ft = 15 ft

***THE FOLLOWING ABBREVIATED TRIGONOMETRY TABLES ARE FOR USE IF A SCIENTIFIC CALCULATOR IS UNAVAILABLE.**

Sine and Cosine of Angles 1 to 45 degrees								
DEG	Sine	Cosine	DEG	Sine	Cosine	DEG	Sine	Cosine
1	0.0175	0.9998	16	0.2756	0.9613	31	0.5150	0.8572
2	0.0349	0.9994	17	0.2924	0.9563	32	0.5299	0.8480
3	0.0523	0.9986	18	0.3090	0.9511	33	0.5446	0.8387
4	0.0698	0.9976	19	0.3256	0.9455	34	0.5592	0.8290
5	0.0872	0.9962	20	0.3420	0.9397	35	0.5736	0.8192
6	0.1045	0.9945	21	0.3584	0.9336	36	0.5878	0.8090
7	0.1219	0.9925	22	0.3746	0.9272	37	0.6018	0.7986
8	0.1392	0.9903	23	0.3907	0.9205	38	0.6157	0.7880
9	0.1564	0.9877	24	0.4067	0.9135	39	0.6293	0.7771
10	0.1736	0.9848	25	0.4226	0.9063	40	0.6428	0.7660
11	0.1908	0.9816	26	0.4384	0.8988	41	0.6561	0.7547
12	0.2079	0.9781	27	0.4540	0.8910	42	0.6691	0.7431
13	0.2250	0.9744	28	0.4695	0.8829	43	0.6820	0.7314
14	0.2419	0.9703	29	0.4848	0.8746	44	0.6947	0.7193
15	0.2588	0.9659	30	0.5000	0.8660	45	0.7071	0.7071

Tangent of Angles 1 - 45 degrees					
DEG	Tan	DEG	Tan	DEG	Tan
1	0.0175	16	0.2867	31	0.6009
2	0.0349	17	0.3057	32	0.6249
3	0.0524	18	0.3249	33	0.6494
4	0.0699	19	0.3443	34	0.6745
5	0.0875	20	0.3640	35	0.7002
6	0.1051	21	0.3839	36	0.7265
7	0.1228	22	0.4040	37	0.7536
8	0.1405	23	0.4245	38	0.7813
9	0.1584	24	0.4452	39	0.8098
10	0.1763	25	0.4663	40	0.8391
11	0.1944	26	0.4877	41	0.8693
12	0.2126	27	0.5095	42	0.9004
13	0.2309	28	0.5317	43	0.9325
14	0.2493	29	0.5543	44	0.9657
15	0.2679	30	0.5774	45	1.0000